Problem Set 1

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1 Problem 1

My age in decimal notation is 17. Converting to base 2, we note that

$$17 = 1 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0.$$

This means that $17_{10} = 10001_2$ and thus our final answer is $\boxed{10001}$.

2 Problem 2

From problem 1, note that since 5 bits were used to express my age in base 2, we would need 5 qubits to express the same value in a superposition state. Moving forward, I use Bra-ket notation with $|0\rangle = 00000$, $|1\rangle = 00001$, $|2\rangle = 00010, \ldots$, $|31\rangle = 11111$.

However, since there is only one classical state of interest (10001), we can set this probability to 1 and the probability of all other classical states to 0. Therefore, our final answer is $|\psi\rangle = 1\,|17\rangle$.

Remark 2.1. *I'm pretty certain I did this question wrong as the probability of measuring each classical state was not equal. I considered a few different alternatives including using gates to combine qubits to achieve* $|10001\rangle$*, but I could not find a solution that way. I also tried solving it by using 6 qubits and giving each of the first 34 qubits a weighting of* $\frac{1}{\sqrt{34}}$*, but that would also violate the rule as the other 30 qubits would have a weighting of 0 and thus not achieve the required output. I believe I am misunderstanding this question at a conceptual level, so hopefully we can cover this further in-depth next Tuesday.*

3 Problem 3/4

Given the superposition $|\psi\rangle = \frac{1}{\sqrt{4}}|0\rangle + \frac{\sqrt{3}}{\sqrt{4}}|1\rangle$, note that the probability of measuring $|1\rangle$ can be computed by solving $|\langle 1|\psi\rangle|^2$:

$$\begin{aligned}\langle 1|\psi\rangle &= \frac{1}{\sqrt{4}}\langle 1|0\rangle + \frac{\sqrt{3}}{\sqrt{4}}\langle 1|1\rangle \\ &= \frac{1}{\sqrt{4}}\cdot 0 + \frac{\sqrt{3}}{\sqrt{4}}\cdot 1 \\ &= \frac{\sqrt{3}}{\sqrt{4}} \\ |\langle 1|\psi\rangle|^2 &= \boxed{\frac{3}{4}}\end{aligned}$$

This is supported by our Qiskit simulation (as seen in figure 1).

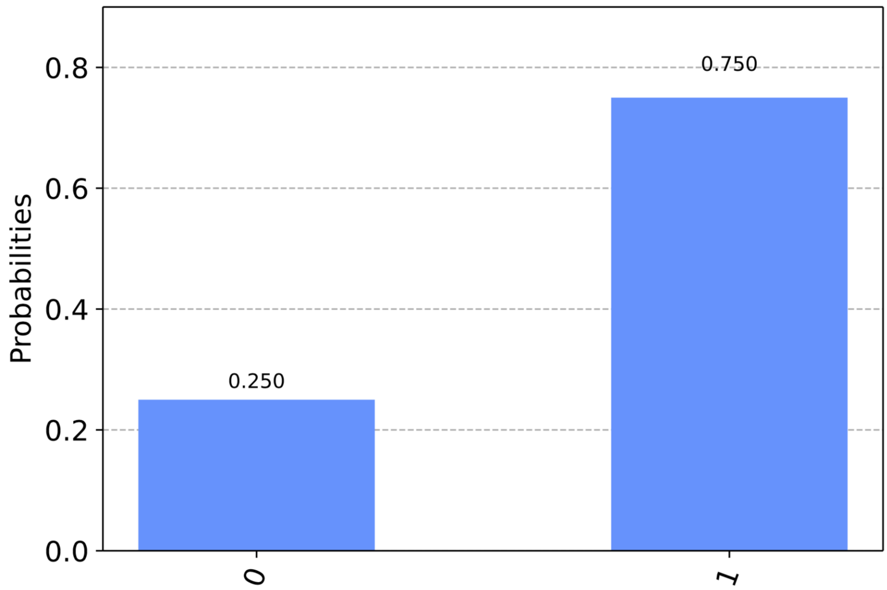

Figure 1: Probabilities of measuring each classical state

4 Problem 5

Applying the same technique showcased in the last section, note that we are attempting to solve $|\langle 1|\psi\rangle|^2$ (since $|01\rangle = |1\rangle$), but since $|1\rangle$ has been initialized to 0 (as there is no coefficient), the probability of measuring the $|01\rangle$ state is just $\boxed{0}$.

5 Problem 6

5.1 1.2 part a

Since $|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}$ and $|1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$, $|0\rangle\,|1\rangle = \begin{bmatrix}1 \times \begin{bmatrix}0\\1\end{bmatrix}\\ 0 \times \begin{bmatrix}0\\1\end{bmatrix}\end{bmatrix} = \begin{bmatrix}0\\1\\0\\0\end{bmatrix}$

$$\text{Statevector} = \begin{bmatrix}0 & 1 & 0 & 0\end{bmatrix}$$

Figure 2: Statevector Output of $|0\rangle\,|1\rangle$ in Jupyter Notebook

5.2 1.2 part b

Since $|+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$, $|0\rangle\,|+\rangle = \begin{bmatrix}1 \times \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}\\ 0 \times \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\\0\\0\end{bmatrix}$

$$\text{Statevector} = \begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0\end{bmatrix}$$

Figure 3: Statevector Output of $|0\rangle\,|+\rangle$in Jupyter Notebook

5.3 1.2 part c

$|+\rangle\,|1\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 \times \begin{bmatrix}0\\1\end{bmatrix}\\ 1 \times \begin{bmatrix}0\\1\end{bmatrix}\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\0\\1\end{bmatrix}$

$$\text{Statevector} = \begin{bmatrix}0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}}\end{bmatrix}$$

Figure 4: Statevector Output of $|+\rangle\,|1\rangle$ in Jupyter Notebook

Remark 5.1. *I find it interesting how, when writing these operations into code, you conduct your operations on qubits from right to left, which was the reverse direction of how I initially expected them to operate. Hopefully we can discuss this during next class, as I fail to understand why this works the way it does.*