Problem Set 2

Tyler King

July 12, 2022

1 Problem 1

Begin by noting that any qubit state must be a superposition of $|0\rangle$ and $|1\rangle$. This means that, if we prove the order of gates HZH is equivalent to an X-gate for these two particular qubit states, all qubit states must be equivalent between X-gates and HZH gates since the $|0\rangle$ and $|1\rangle$ states serve as an orthogonal basis for all superpositions.

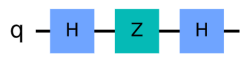


Figure 1: Sequence of gates HZH

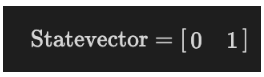


Figure 2: Result of gates HZH

Figure 1 yields the statevector observed in figure 2, which is clearly equivalent to applying an X-gate on $|0\rangle$.

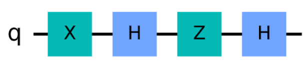


Figure 3: Sequence of gates XHZH

Similarly, figure 3 yields the statevector observed in figure 4, which is clearly equivalent to applying an X-gate on $|1\rangle$. Since the two particular cases have been proven to be equivalent between X-gates and HZH gates, we can conclude that all qubits where HZH gates were applied are equivalent to the same qubit with an X-gate applied.

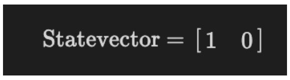


Figure 4: Result of gates XHZH

2 Problem 2

By simple matrix multiplication, note that multiplying the X-gate and Z-gate yields:

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix},$$

which is just a multiple of i off of the Y-gate. Therefore, we can make the statement that the Y-gate is equivalent to applying Z and then X gates and multiplying it by i.

Remark 2.1. *I believe the multiply by i component is ignored because of the statement that says "ignore global phase".*

When we apply these gates in Qiskit, we observe similar results to the ones outlined earlier. Again, by noting that all qubit states are a superposition of qubits $|0\rangle$ and $|1\rangle$, we proceed to prove that these two qubits are equivalent when undergoing the Y-gate or the Z/X-gates in succession (figures not included for clarity, but they confirm the statements proposed in this section).

3 Problem 3

The concept that time is money is frequently used to portray the importance of efficiency, and this concept holds true when applying it to algorithms. Standard digital computers are frequently less efficient than quantum computers simply because they depend on binary states (0s or 1s) while quantum computers are able to be in superposition (a combination of these two states simultaneously). This radically different technique allows quantum computers to operate under unique rules. As such, there is a rapid reduction in complexity (a way to measure the runtime of a certain algorithm) for certain cases such as factoring numbers showcases its potential compared to standard computational algorithms and is a big reason why people look forward to seeing the growth of this field.

4 Problem 4

Begin by noting that

$$\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}.$$

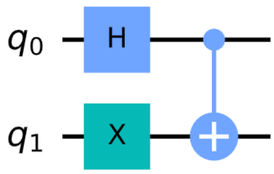
Working backwards, we can note that the CNOT gate (when the second qubit is the constant) switches the amplitudes of $|01\rangle$ and $|11\rangle$, which means that when

$$|a\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \text{CNOT}\,|a\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}.$$

Therefore, out goal is to observe a qubit superposition of

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix},$$

which can be computed by $|1\rangle \bigotimes |+\rangle$. Therefore, our final circuit is:



(confirmed using Qiskit).